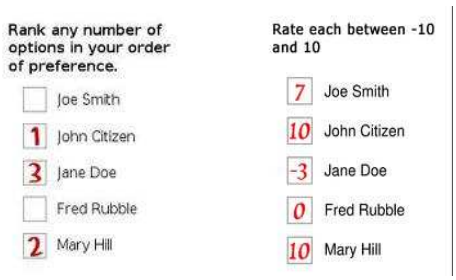
11.1 Voting Methods



Voting Methods are the different systems which can be used to give scores to candidates and select the winner.

When there are two candidates, they all give the same outcome. But when there are three or more candidates, they can have drastically different outcomes.

DEFINITION **The Plurality Method**
Each person votes for his or her favorite candidate. The candidate receiving the most votes is declared the winner.

This is the method the U.S. uses.

Example: 33 people vote for 4 candidates.

Ann	10
Ben	9
Carim	11
Doreen	3

Who wins?

Ann	10
Ben	9
Carim	11
Doreen	3

Because Carim has the most votes, he is declared the winner.

In the Borda count method, voters rank candidates on their ballots. Such a ballot is called a *preference ballot*.

Votes are tallied and identical ballots are grouped in a table called a *preference table*.

DEFINITION **The Borda Count Method**
If there are k candidates in an election, each voter ranks all candidates on the ballot. Then the first choice is given k points, the second choice is given $k-1$ points, the third choice $k-2$ points, and so on.* The candidate who receives the most total points wins the election.

Example of one vote:

1st	C
2nd	A
3rd	B
4th	D

C receives 4 points
A receives 3 points
B receives 2 points
D receives 1 point

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Total Points (1st place gets 3 points, etc)
A = 3 + 2 + 2 + 1 + 1 = 9
B = 2 + 3 + 1 + 3 + 2 = 11
C = 1 + 1 + 3 + 2 + 3 = 10

B has the most points and thus wins.

Example 3 candidates, 6 voters.

Voter 1: A B C
Voter 2: C B A
Voter 3: B C A
Voter 4: A B C
Voter 5: C B A
Voter 6: C B A

Example 3 candidates, 6 voters.

Voter 1: A B C
Voter 2: C B A
Voter 3: B C A
Voter 4: A B C
Voter 5: C B A
Voter 6: C B A

Idea: count how often each candidate is in 1st 2nd and 3rd place and multiply by the points they would get.

	1st	2nd	3rd
A	2	0	4
B	1	5	0
C	3	1	2

Total Points
A = 2x3 + 0x2 + 4x1 = 10
B = 1x3 + 5x2 + 0x1 = 13
C = 3x3 + 1x2 + 2x1 = 13
Tie between B and C.

Example with the same 33 votes and 4 candidates.

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	D
2nd	A	C	D	D	A	A
3rd	B	B	B	B	D	C
4th	D	D	A	C	C	B

Carim, Ann, Ben, and Doreen have the same number of first-place votes as before.

Ann has 18 second-place votes.

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	D
2nd	A	C	D	D	A	A
3rd	B	B	B	B	D	C
4th	D	D	A	C	C	B

Carim, Ann, Ben, and Doreen have the same number of first-place votes as before.

Ann has 18 second-place votes.

Counting up Ann's votes:

$$10 \times 4 + 18 \times 3 + 0 \times 2 + 5 \times 1 = 40 + 54 + 0 + 5 = 99$$

ten first-place votes times 4 points

eighteen second-place votes times 3 points

five fourth-place votes times 1 point

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	D
2nd	A	C	D	D	A	A
3rd	B	B	B	B	D	C
4th	D	D	A	C	C	B

Carim, Ann, Ben, and Doreen have the same number of first-place votes as before.

Ann has 18 second-place votes.

Candidates	Number of Points				
	1st-Place Votes × 4 (Points)	2nd-Place Votes × 3 (Points)	3rd-Place Votes × 2 (Points)	4th-Place Votes × 1 (Points)	Total Points
A	$10 \times 4 = 40$	$18 \times 3 = 54$	$0 \times 2 = 0$	$5 \times 1 = 5$	99
B	$9 \times 4 = 36$	$0 \times 3 = 0$	$21 \times 2 = 42$	$3 \times 1 = 3$	81
C	$11 \times 4 = 44$	$7 \times 3 = 21$	$3 \times 2 = 6$	$12 \times 1 = 12$	83
D	$3 \times 4 = 12$	$8 \times 3 = 24$	$9 \times 2 = 18$	$13 \times 1 = 13$	67
				Total	330

DEFINITION **The Plurality-with-Elimination Method**

Each voter votes for one candidate. A candidate receiving a majority of votes is declared the winner. If no candidate receives a majority of votes, then the candidate (or candidates) with the fewest votes is dropped from the ballot and a new election is held. This process continues until a candidate receives a majority of votes.

Remove the person with the lowest 1st place votes, and re-rank the votes with one less person. Repeat until 1 remains.

1st election is between all 4 people.
2nd election is between top 3 people.
3rd election is between top 2 people.

All using the same original ballots.

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Example 3 candidates, 5 voters.

Voter 1: A B C	|	B C	| B
Voter 2: B A C	|	B C	| B
Voter 3: C A B	|	C B	| B
Voter 4: B C A	|	B C	| B
Voter 5: C B A	|	C B	| B
A has least 1st place votes. Remove A.	|	C has least 1st place votes Remove C	| B wins

Example 3 candidates, 6 voters.

Voter 1: A B C
Voter 2: C B A
Voter 3: B C A
Voter 4: A B C
Voter 5: C B A
Voter 6: C B A

Example 3 candidates, 6 voters.

Voter 1: A B C	|	A C	| C
Voter 2: C B A	|	C A	| C
Voter 3: B C A	|	C A	| C
Voter 4: A B C	|	A C	| C
Voter 5: C B A	|	C A	| C
Voter 6: C B A	|	C A	| C
Remove B	|	Remove A	| C wins

Same example with 33 voters.

Cross out the looser.

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	D
2nd	A	C	D	D	A	A
3rd	B	B	B	B	D	C
4th	D	D	A	C	C	B

Carim, Ann, Ben, and Doreen have the same number of first-place votes as before.

Ann has 18 second-place votes.

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	D
2nd	A	C	D	D	A	A
3rd	B	B	B	B	D	C
4th	D	D	A	C	C	B

Carim, Ann, Ben, and Doreen have the same number of first-place votes as before.

Ann has 18 second-place votes.

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	~~D~~
2nd	A	C	~~D~~	~~D~~	A	A
3rd	B	B	B	B	~~D~~	C
4th	~~D~~	~~D~~	A	C	C	B

Doreen has the fewest first-place votes, so she is eliminated.

Remaining candidates move up.

Preference	Number of Ballots					
	6	7	5	3	9	3
1st	C	A	C	A	B	~~D~~
2nd	A	C	~~D~~	~~D~~	A	A
3rd	B	B	B	B	~~D~~	C
4th	~~D~~	~~D~~	A	C	C	B

Doreen has the fewest first-place votes, so she is eliminated.

Remaining candidates move up.

Preference	Number of Ballots				
	6	10	5	3	9
1st	C	A	C	A	~~B~~
2nd	A	C	~~B~~	~~B~~	A
3rd	~~B~~	~~B~~	A	C	C

Ben now has the fewest first-place votes, so he is eliminated.

Preference	Number of Ballots				
	6	10	5	3	9
1st	C	A	C	A	~~B~~
2nd	A	C	~~B~~	~~B~~	A
3rd	~~B~~	~~B~~	A	C	C

Ben now has the fewest first-place votes, so he is eliminated.

Preference	Number of Ballots				
	6	10	5	3	9
1st	C	A	C	A	A
2nd	A	C	A	C	C

Ann has 22 first-place votes; Carim has 11.

DEFINITION **The Pairwise Comparison Method**
Voters first rank all candidates. If A and B are a pair of candidates, we count how many voters prefer A to B and vice versa. Whichever candidate is preferred the most receives 1 point. If A and B are tied, then each receives $\frac{1}{2}$ point. Do this comparison, assigning points, for each pair of candidates. At the end, the candidate receiving the most points is the winner.

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

Example 3 candidates, 5 voters.

Voter 1: A B C
Voter 2: B A C
Voter 3: C A B
Voter 4: B C A
Voter 5: C B A

A-B pair
A beats B 2 times
B beats A 3 times
B gets 1 point
A-C pair
C beats A 3 to 2
C gets 1 point
B-C pair
B beats C 3 to 2
B gets 1 point

Total points: A = 0 B = 2 C = 1
B wins the most mini-elections

Example 3 candidates, 6 voters.

Voter 1: A B C
Voter 2: C B A
Voter 3: B C A
Voter 4: A B C
Voter 5: C B A
Voter 6: C B A

Example 3 candidates, 6 voters.

Voter 1: A B C
Voter 2: C B A
Voter 3: B C A
Voter 4: A B C
Voter 5: C B A
Voter 6: C B A

A-B pair
B beats A 4 to 2
B gets 1 point
A-C pair
C beats A 4 to 2
C gets 1 point
B-C pair
B and C tie!
B and C get ½ point

Total Points: A = 0 B = 1½ C = 1½
B and C tie.

The Pairwise Comparison Method

• Example: Customers were asked to rank their preferences for (T)acos, (N)achos, and (B)urritos at a restaurant (see table). Using the pairwise comparison method, decide which item is preferred.

	Number of Ballots					
Preference	2,108	864	1,156	1,461	1,587	1,080
1st	T	T	N	N	B	B
2nd	N	B	T	B	T	N
3rd	B	N	B	T	N	T

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The Pairwise Comparison Method

- Solution: We first compare T with N.

1,156 + 1,461 + 1,080 = 3,697 prefer N over T.

Preference	Number of Ballots					
	2,108	864	1,156	1,461	1,587	1,080
1st	T	T	N	N	B	B
2nd	N	B	T	B	T	N
3rd	B	N	B	T	N	T

2,108 + 864 + 1,587 = 4,559 prefer T over N.

We award 1 point to T.

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The Pairwise Comparison Method

We next compare T with B.

1,156 + 1,461 + 1,080 = 3,697 prefer N over T.

Preference	Number of Ballots					
	2,108	864	1,156	1,461	1,587	1,080
1st	T	T	N	N	B	B
2nd	N	B	T	B	T	N
3rd	B	N	B	T	N	T

2,108 + 864 + 1,587 = 4,559 prefer T over N.

T and B each receive one-half point.

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The Pairwise Comparison Method

Finally we compare N with B. We have that 2,108 + 1,156 + 1,461 = 4,725 customers prefer N over B. Also, 864 + 1,587 + 1,080 = 3,531 customers prefer B over N. We award N 1 point.

Preference	Number of Ballots					
	2,108	864	1,156	1,461	1,587	1,080
1st	T	T	N	N	B	B
2nd	N	B	T	B	T	N
3rd	B	N	B	T	N	T

T has 1.5 points, N has 1 point, and B has 0.5 points. T is preferred.